Polarization of the Tagged Compton Backscattered Laser Photons.
Results of Monte Carlo Simulation.

A. S. Omelaenko, Yu. P. Peresun'ko, *

Yu. M. Ranyuk†, I. M. Shapoval,

National Science Center

Kharkov Institute of Physics and Technology,

310108,Kharkov, Ukraine,

and V. G. Nedorezov‡

Kurchatov Institute,123182 , Moscow.

Abstract

Polarization characteristics of the gamma beam obtained by the Compton back scattering of laser photons on high energy electrons are evaluated by Monte-Carlo simulations. It is assumed that outgoing photons are tagged; the energy dispersion of the tagging photons and emittance of the initial electrons are taken into account. Dependence of the final photon polarization parameters on measured photon energy is obtained. It is shown that polarization of final photons is decreasing with change for the worth of the tagging energy resolution. Calculations have been applied for the storage ring SIBERIA-2 at Kurchatov Institute. The obtained results indicate a reasonability for construction of gamma-polarimeters on existing and planned facilities for the on-line measurement of the final photon beam polarization parameters.

1   Theoretical description of the process of Compton scattering

Method of obtaining monochromatic and polarized photon beams of high energy and intensity by the Compton back scattering of laser photon on high-energy electron beam is widely used in many active and planned facilities [1, 2]. Idea of using the Compton back scattering of laser photons on relativistic electron beams as source of monochromatic and polarized gamma - radiation was specified in works of Harutyunian and Tumanian[3, 4] and in work of Milborn[5], although polarization phenomenons in this reaction were considered earlier[6]. Authors[3, 4] used in their evaluations formula for Stoke's parameters of final photon from monograph[7] which contained unfaithful expression for Stoke's parameter $\xi_2^{(f)}$ of final photon circular polarization. In publishing 1969 of this monograph without

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commentaries was brought correct expression for this value. However, on the strength of popularity of works [3, 4], casus with parameter $\xi_2^{(f)}$ is discussed recently[8].

Important element in practical using of considered process is a tagging system, which allows to measure an energy of outgoing photons. Influence of this system on polarization features of final photon beam hitherto not explored sufficiently. The purpose of this work is the detail analysis of the polarization characteristics of the outgoing photon beam and studying of influence on it of the different parameters - the electron beam divergence, the uncertainties in definition of the outgoing photon energy by tagging of the final electrons and spin characteristics of initial electron beam.
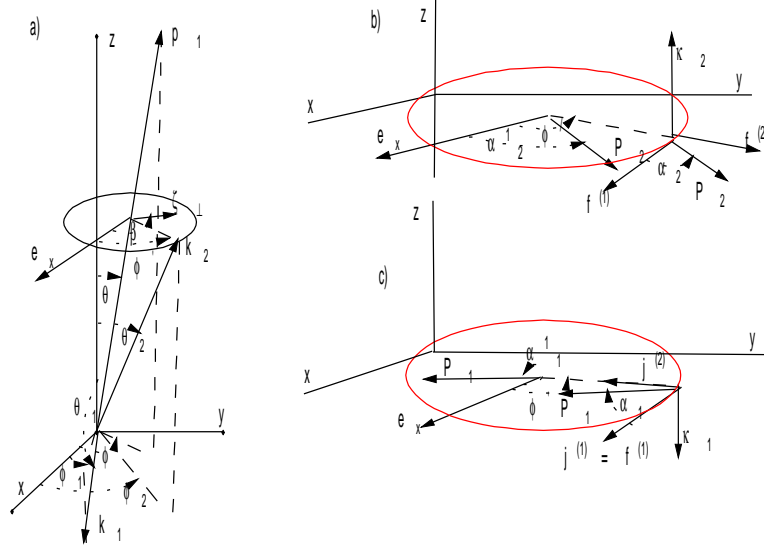

Figure 1:

Let's consider the kinematics of the process, that is shown in Figure 1. In the laboratory frame where axis z coincides with a beam's axis and x axis belongs to horizontal plane of accelerator's beam initial electron with a momentum $\vec{p}_1$ (polar angle — θ, azimuthal angle — φ) interacts with a laser photon with a momentum $\vec{k}_1$ (polar angle — θ_1, azimuthal angle — φ_1). Scattered photon has the 3–momentum $\vec{k}_2$ (polar angle — θ_2, azimuthal angle — φ_2). Momentum of the final electron $\vec{p}_2$ is not shown. From 4–momentum conservation law $k_1 + p_1 = k_2 + p_2$ follows $(k_2 \cdot (p_1 + k_1)) = (k_1 \cdot p_1)$, or

$$\omega_2\{1 - v \cdot [\cos\theta \cdot \cos\theta_2 + \sin\theta \cdot \sin\theta_2 \cos(\varphi_2 - \varphi)]$$
$$+ \frac{\omega_1}{\varepsilon_1} \cdot [1 - \cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2 \cdot \cos(\varphi_1 - \varphi_2)]\} \qquad (1)$$
$$= \omega_1 \cdot \{1 - v \cdot [\cos\theta \cdot \cos\theta_1 + \sin\theta \cdot \sin\theta_1 \cdot \cos(\varphi_1 - \varphi)]\}$$

where ω_1, ω_2 — energies of the initial and final photons, ε_1 — energy of initial electron; $v = \sqrt{1 - m^2/\varepsilon_1^2}$ — velocity of initial electron. The angles θ, φ of the initial electron are random values. Usually are considered values $\theta_x = \theta\cos\varphi$ and $\theta_y = \theta\sin\varphi$ that supposed to be normally distributed with a means $\theta_x^o = 0$ and $\theta_y^o = 0$ and variances σ_x and σ_y correspondingly. Electron beams of the modern storage rings have $\sigma_x \sim 10^{-3} \div 10^{-4}$ rad, $\sigma_y \sim 10^{-4} \div 10^{-5}$ rad. The range of allowed angles of final photon θ_2, φ_2 is determined by the collimator's size and is of the order $10^{-3} - 10^{-4}$ rad.

So we can with a good accuracy change in (1):

$$\sin\theta \cdot \cos\varphi \to \theta_x, \quad \sin\theta \cdot \sin\varphi \to \theta_y, \quad \sin\theta_2 \to \theta_2,$$

2

$$\cos\theta \to 1 - \frac{1}{2}\theta^2, \quad \cos\theta_2 \to 1 - \frac{1}{2}\theta_2^2.$$

Replacing variables

$$[\theta_2, \varphi_2] \to [\alpha_x = \theta_2 \cos\varphi_2, \quad \alpha_y = \theta_2 \sin\varphi_2],$$

we can write (1) in the form:

$$\left(\alpha_x - \theta_x - \frac{\omega_1}{\varepsilon_1}\sin\theta_1 \cos\varphi_1\right)^2 + \left(\alpha_y - \theta_y - \frac{\omega_1}{\varepsilon_1}\sin\theta \sin\varphi_1\right)^2 = r^2 \tag{2}$$

where

$$r^2 = \gamma^{-2}\left[x\left(\frac{\varepsilon_1 - \omega_2}{\omega_2}\right) - 1\right]; \qquad \gamma = \frac{\varepsilon_1}{m}; \qquad x = 2\frac{(k_1 \cdot p_1)}{m^2}$$

Equation (2) is valid with an accuracy up to terms

$$\left[\frac{\omega_1}{\varepsilon_1}, \quad \theta_2^2, \quad \theta_x^2, \quad \theta_y^2, \quad \gamma^{-2}\right] \ll 1$$

Thus we can see from (2) that photons with a definite energy ω_2 are emitted at the surface of the circle cone with an axes along direction of the vector $\vec{l} = \vec{n} + \frac{\omega_1}{\varepsilon_1}\vec{\kappa}_1$ and the opening angle r $(\vec{n} = \vec{p}_1/|\vec{p}_1|, \vec{\kappa}_1 = \vec{k}_1/\left|\vec{k}_1\right|)$

From the condition of positive definition of r^2 maximal possible energy of secondary photon is

$$\omega_{2\max} = \varepsilon_1 \frac{x}{1+x} \tag{3}$$

An invariant form of the cross section of the Compton scattering of the polarized photon by the polarized electron when one detect final photon with a Stoke's parameters $\widetilde{\xi}_i^{(f)}$ is [7, 10]:

$$\frac{d^2\sigma}{dy d\varphi_2} = Sp\{\widehat{\rho}_c \cdot \widehat{\rho}^{(f)}\} = \frac{r_o^2}{2x^2}\left\{F_0 + \widetilde{\xi}_1^{(f)}F_1 + \widetilde{\xi}_2^{(f)}F_2 + \widetilde{\xi}_3^{(f)}F_3\right\}; \tag{4}$$

where

$$\widehat{\rho}_c = \frac{r_o^2}{2x^2}\left(\begin{array}{cc} F_0 + F_3 & F_1 + iF_2 \\ F_1 - iF_2 & F_0 - F_3 \end{array}\right);$$

$$\widehat{\rho}^{(f)} = \frac{1}{2}\left(\begin{array}{cc} 1 + \widetilde{\xi}_3^{(f)} & \widetilde{\xi}_1^{(f)} + i\widetilde{\xi}_2^{(f)} \\ \widetilde{\xi}_1^{(f)} - i\widetilde{\xi}_2^{(f)} & 1 - \widetilde{\xi}_3^{(f)} \end{array}\right);$$

$$F_0 = \frac{x}{y} + 4z(1+z)\left(1 - \widetilde{\xi}_3^{(1)}\right) - 2\widetilde{\xi}_2^{(1)}\, z\left[(1 + 2\,z)(s \cdot k_1) + (s \cdot k_2)\right];$$

$$F_1 = 2\widetilde{\xi}_1^{(1)}\,(1 + 2z) - 4\widetilde{\xi}_2^{(1)}\frac{z}{y}\,(sk_1p_1k_2)\,;$$

3

$$F_2 = \widetilde{\xi}_2^{(1)}(1+2z)\left(\frac{x}{y}+\frac{y}{x}\right) - 2z[(s\cdot k_1) + (1+2z)(s\cdot k_2)]$$
$$+2\widetilde{\xi}_3^{(1)}z[(1+2z)(s\cdot k_2) - \frac{y}{x}(s\cdot k_1)] + 4\widetilde{\xi}_1^{(1)}\frac{z}{x}(sk_1p_1k_2);$$

$$F_3 = -4z1 + z\left(1 - \widetilde{\xi}_3^{(1)}\right) + 2\widetilde{\xi}_3^{(1)} + 2\widetilde{\xi}_2^{(1)}z[(1+2z)(s\cdot k_1) - \frac{x}{y}(s\cdot k_2)];$$

$$x = 2\frac{(k_1\cdot p_1)}{m^2}, \quad y = 2\frac{(k_2\cdot p_1)}{m^2}, \quad z = \frac{1}{x} - \frac{1}{y};$$

s_μ — 4–vector of the initial electron polarization,
$(sk_1p_1k_2) = \varepsilon^{\mu\nu\rho\sigma}s_\mu k_{1\nu}p_{1\rho}k_{2\sigma}/m^3, \quad \varepsilon^{0123} = +1$.
r_o—classical radius of electron, φ_2–azimuthal angle of the final photon (see figure 1).

Here Stoke's parameters of the initial photon $\widetilde{\xi}_i^{(1)}$ are defined relatively to the relativistic invariant unit vectors [9, 10]:

$$j_\mu^{(1)} = \varepsilon_{\mu\nu\rho\sigma}k_{1\nu}p_{1\rho}k_{2\sigma}/a, \qquad j_\mu^{(2)} = \frac{2}{m^2x}\varepsilon_{\mu\nu\rho\sigma}k_{1\nu}p_{1\rho}j_\sigma^{(1)}; \tag{5}$$

where
$a = \frac{m^3}{2}\left[(x-y)(xy+y-x)\right]^{1/2}$,

and Stoke's parameters of the final photon, $\widetilde{\xi}_i^{(f)}$,- to the unit vectors:

$$f_\mu^{(1)} = j_\mu^{(1)}; \qquad f_\mu^{(2)} = 2/\left(m^2y\right)\varepsilon_{\mu\nu\rho\sigma}k_{2\nu}p_{1\rho}f_\sigma^{(1)} \tag{6}$$

we shall use following parametrization of the photon's polarization characteristics [11]:

$$\xi_1 = P\cdot\cos 2\beta\cdot\sin 2\alpha; \qquad \xi_3 = P\cdot\cos 2\beta\cdot\cos 2\alpha; \qquad \xi_2 = P\cdot\sin 2\beta \tag{7}$$

where $P = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$–degree of the total polarization,
$P_l = \sqrt{\xi_1^2 + \xi_3^2}$–degree of the linear photon polarization,
α –the angle between direction of axis $\vec{j}^{(1)}$ and direction of maximal linear polarization of photon that is reading counter clockwise when one looks from the end of photon momentum vector.

Gauge transformation permits to choose unit vectors $j_\mu^{(i)}$, $f_\mu^{(i)}$ as a pure space vectors that are orthogonal to a $\vec{k}_1$ and $\vec{k}_2$ correspondingly.

Such a vectors, $\vec{j}_\perp^{(1)}$, $\vec{j}_\perp^{(2)}$ and $\vec{f}_\perp^{(1)}$, $\vec{f}_\perp^{(2)}$ are calculated in Appendix for the case of arbitrary angles of initial photon θ_1, φ_1 with an accuracy up to a small terms of the order of θ^2, θ_2^2, γ^{-2} and have a form:

$$\vec{j}_\perp^{(1)} = \vec{e}_x\left[\sin\varphi' + (1+\cos\theta_1)\cos\varphi_1\sin(\varphi_1-\varphi')\right] \tag{8}$$
$$+\vec{e}_y\left[-\cos\varphi' + (1+\cos\theta_1)\sin\varphi_1\sin(\varphi_1-\varphi')\right]$$
$$-\vec{e}_z\sin\theta_1\sin(\varphi_1-\varphi');$$

$$\vec{j}_\perp^{(2)} = \vec{e}_x\left[\cos\theta_1\cos\varphi_1 + (1+\cos\theta_1)\Pr\sin\varphi_1\sin(\varphi_1-\varphi')\right] \tag{9}$$
$$+\vec{e}_y\left[\cos\theta_1\sin\varphi' + (1+\cos\theta_1)\cos\varphi_1\sin(\varphi_1-\varphi')\right]$$
$$-\vec{e}_z\sin\theta_1\cos(\varphi_1-\varphi');$$

4

$$\vec{f}_\perp^{(1)} = \vec{e}_x \sin \varphi' - \vec{e}_y \cos \varphi'; \tag{10}$$

$$\vec{f}_\perp^{(2)} = \vec{e}_x \sin \varphi' + \vec{e}_y \cos \varphi'; \tag{11}$$

Here φ' is the azimuthal angle on the surface of the cone of the final photon emitting. This angle is reading from $\vec{e}_x$ when direction of $\vec{p}_1$ serves as a polar axis (see figure 1). Expressions (8 - 11) are not valid, of course, in a very narrow interval of final photon energy ω_2 near maximal final photon energy $\omega_{2\,\mathrm{max}}$ when defined in equation (2) value $r = \gamma^{-1}[(1-x)(\omega_{2\,\mathrm{max}} - \omega_2)/\omega_2]^{1/2}$ becomes less or order of $\{\gamma^{-2},\ \theta^2,\ \theta_2^2\}$. In such interval of energies of final photon ω_2 linear polarization of outgoing photon may be strongly correlated with directions of initial photon and electron moments, but we shall not discuss this point in the current work.

On the experiment polarization of photons is determined relatively to the fixed axes $\{\vec{e}_x,\ \vec{e}_y,\ \vec{e}_z\}$ of the laboratory frame. In this frame Stoke's parameters of photons are $\xi_i^{(1)}$, $\xi_i^{(f)}$ and can be expressed in terms of $\widetilde{\xi}_i^{(1)}$, $\widetilde{\xi}_i^{(f)}$ by the following way:

Let accordingly to parametrization (7) vector of linear polarization of outgoing photon is $\vec{P}_2$ and has an angle α_2 with an axes $\vec{f}_\perp^{(i)}$ (see Fig. 1). Then it's angle $\widetilde{\alpha}_2$ with an axes $\vec{e}_x$ is $\widetilde{\alpha}_2 = \alpha_2 + (\varphi' - \frac{\pi}{2})$ and we can write:

$$
\begin{aligned}
\xi_1^{(f)} &= P_2 \cos 2\beta_2 \sin 2\widetilde{\alpha}_2 = P_2 \cos 2\beta_2 \sin 2(\alpha_2 + \varphi' - \frac{\pi}{2}) \\
&= -\widetilde{\xi}_1^{(f)} \cos 2\varphi' - \widetilde{\xi}_3^{(f)} \sin 2\varphi';
\end{aligned} \tag{12}
$$

$$\xi_3^{(f)} = -\widetilde{\xi}_3^{(f)} \cos 2\varphi' + \widetilde{\xi}_1^{(f)} \sin 2\varphi'; \tag{13}$$

$$\xi_2^{(f)} = \widetilde{\xi}_2^{(f)} \tag{14}$$

And for initial photon $\widetilde{\alpha_1} = \alpha_1 - (\varphi' - \frac{\pi}{2})$ (angles α_1 and φ' are reading in opposite directions), therefore[1]:

$$\xi_1^{(1)} = -\widetilde{\xi}_1^{(1)} \cos 2\varphi' + \widetilde{\xi}_3^{(1)} \sin 2\varphi'; \tag{15}$$

$$\xi_3^{(1)} = -\widetilde{\xi}_3^{(1)} \cos 2\varphi' - \widetilde{\xi}_1^{(1)} \sin 2\varphi'; \tag{16}$$

$$\widetilde{\xi}_2^{(1)} = \xi_2^{(1)}. \tag{17}$$

Here and below we put $\theta_1 = \pi$. It is easy to see that uncertainties in determination of θ_1 do not affect on polarization characteristics of outgoing photon up to terms of order $(\Delta\theta_1)^2$.

Thus one can see that natural and convenient variables for describing of considered process of Compton backscattering of laser photon on an relativistic electron with a tagging of final photons when on experiment is straightforward determined the energy of

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[1]In this point work [9] contains mistake. Besides that in expression for cros section in [9] is missing factor 1/2. Other formula of [9] which we used have been checked and are correct.

this photon ω_2 are variables $u = \gamma r$ and φ'. In this variables after substitution of (12–17) considered cross section has a form (see [9]):

$$\frac{d^2\sigma}{d\omega_2\, d\varphi'} = \frac{r_o^2}{2\varepsilon_1 x \left(1+u^2\right)^2 \left(1+x+u^2\right)} \left\{ \Phi_0 + \widetilde{\xi}_1^{(f)} \Phi_1 + \widetilde{\xi}_2^{(f)} \Phi_2 + \widetilde{\xi}_3^{(f)} \Phi_3 \right\} \qquad (18)$$

where

$$\begin{aligned}
\Phi_0 &= 2 + 2x + x^2 + u^2\left(2+x^2\right) + 2u^4\left(1+x\right) + 2u^6 \\
&\quad -4\left(\xi_3^{(1)}\cos 2\varphi' - \xi_1^{(1)}\sin 2\varphi'\right) u^2 \left(1+x+u^2\right) \qquad (19) \\
&\quad -\xi_2^{(1)}[\lambda x\left(1-u^2\right)\left(2+x+2u^2\right) + 2\zeta_\perp \cos\left(\beta-\varphi'\right) xu\left(1+u^2\right)];
\end{aligned}$$

$$\begin{aligned}
\Phi_1 &= 2(1+x+u^2)(\xi_1^{(1)}(-1+u^4\cos 4\varphi') + u^4\xi_3^{(1)}\sin 4\varphi' \\
&\quad -2u^2\sin 2\varphi' + \xi_2^{(1)}\varsigma_\perp xu\sin(3\varphi'-\beta)); \qquad (20)
\end{aligned}$$

$$\begin{aligned}
\Phi_2 &= -\xi_2^{(1)}\left(1-u^2\right)\left(2+2x+x^2+2u^2\left(2+x+u^2\right)\right) \\
&\quad +\lambda x\left(2+x+xu^2+2u^4\right) + 4\left(-\xi_3^{(1)}\cos 2\varphi' + \xi_1^{(1)}\sin 2\varphi'\right)\lambda xu^2 \\
&\quad +2\left(1+\xi_3^{(1)}\cos 2\varphi' - \xi_1^{(1)}\sin 2\varphi'\right)\zeta_\perp xu\left(1-u^2\right)\cos\left(\beta-\varphi'\right) \\
&\quad +2\left(\xi_1^{(1)}\cos 2\varphi' + \xi_3^{(1)}\sin 2\varphi'\right)\zeta_\perp \sin\left(\beta-\varphi'\right) xu\left(1+u^2\right); \qquad (21)
\end{aligned}$$

$$\begin{aligned}
\Phi_3 &= 2\left(1+x+u^2\right)[\xi_3^{(1)}(1+u^4\cos 4\varphi') - \xi_1^{(1)}u^4\sin 4\varphi' \\
&\quad -2u^2\cos 2\varphi' + \xi_2^{(1)}\zeta_\perp xu\cos\left(\beta-\varphi'\right)]; \qquad (22)
\end{aligned}$$

Here

$$u^2 = (\gamma r)^2 = (1+x)\frac{\omega_{2\max}-\omega_2}{\omega_2};$$

λ–longitudinal polarization of initial electron, $\xi_\perp$ -transverses polarization of initial electron, β- angle between x axes and direction of transverses polarization of electron.

According to general theory from (18-22) follows that proper Stoke's parameters of outgoing photon determined relatively to the laboratory frame axes $\{x,y,z\}$ are:

$$\xi_1^{(2)} = \frac{\Phi_1}{\Phi_0}, \; \xi_2^{(2)} = \frac{\Phi_2}{\Phi_0}, \; \xi_3^{(2)} = \frac{\Phi_3}{\Phi_0} \qquad (23)$$

It would be mentioned that in realistic experiment of Compton backscattering of laser photons when initial relativistic electrons have some angle dispersion there is not existing one–to–one correspondence between direction of emitting of final photon and it's polarization characteristics. Actually, photon with defined energy ω_2 and angles θ_2, φ_2 (see figure 1) may be emitted by the whole set of initial electrons that have direction of motion situated at the surface of cone with corresponding to ω_2 angle of opening r.

Corresponding to each such event angle φ' is different and therefore will be different polarization of emitted photon. Hence, we have to consider only averaged over all possible events of Compton interaction polarization characteristics of outgoing photon. We should like to emphasize that for calculation of different averaging polarization parameters (for example $\left\langle \xi_i^{(2)} \right\rangle$), one has to average values Φ_i and to build $\left\langle \xi_i^{(2)} \right\rangle$ by using values $\langle \Phi_i \rangle$.

2 Some details of calculations.

It follows from the previous consideration that the event generator for the Monte-Carlo simulation of the process of Compton backscattering of laser photon by accelerated electron beam when outgoing photon beam is tagged and collimated has to consist of following parts:

1. Generator of random numbers for the values ω_2, φ' with appropriate distribution;

2. Although values which describe direction of initial electron motion, do not enter to expressions (18) it could effect on distribution of ω_2, φ' through the condition of final photon passing across the collimator with radius R situated at distance L from the Compton interaction point:

$$\left(\theta_x + r\cos\varphi'\right)^2 + \left(\theta_y + r\sin\varphi'\right)^2 \leq (R/L)^2 \tag{24}$$

So we have to include appropriate generator for this values. Distribution of this values is well known – with a good accuracy it described by the normal distribution with mean $\theta_x^{(0)} = \theta_y^{(0)} = 0$ and variance σ_x^2, σ_y^2 correspondingly; φ' - uniformly distributed in the range $[0, 2\pi]$, and question about distribution of ω_2 requests some discussion. As it is known, the procedure of tagging of the photons in the process of Compton backscattering of laser photons on electron beam consists of measuring of energy of the scattered electrons ε_2 that is connected with the energy of other taking part in this process particles by the simple equation $\varepsilon_1 + \omega_1 = \varepsilon_2 + \omega_2$. Energy ε_1 of initial electron and energy ω_1 of laser photon is known with a high accuracy. Energy ε_2 (and therefore - ω_2) usually is measured by the magnet spectrometer with some error. Otherwise speaking, ω_2 is random value that is distributed with some appropriate distribution and result of it's measuring - value of mean ω_{20} of this distribution. Distribution of this value has to be calculated by method of Monte-Carlo simulation for the cases of correspondent facilities. In our calculations should be used such a distributions for the value ω_2. But in the calculations of current article we restrict ourselves by using of the first two known moments of such distributions – the mean ω_{20} and variance σ_ω^2 and have approximated distribution of ω_2 by the modified Gaussian distribution:

$$f(\omega_2) = \frac{A}{\sigma_\omega\sqrt{2\pi}}\exp\{-\frac{(\omega_2 - \omega_{20})^2}{2\sigma_\omega^2}\}\Theta(\omega_2)\Theta(\omega_{2\max} - \omega_2) \tag{25}$$

Where A - an appropriate normalization factor.

That is events that did not belong to the bounded physical region $0 \leq \omega_2 \leq \omega_{2\max}$ has been rejected. Thus, in our calculations we have used an event generator that consists of generator of the set of random numbers $\{\theta_x, \theta_y, \omega_2, \varphi'\}$ with normal distributions for $\{\theta_x, \theta_y\}$, uniformly distributed φ', and distribution (25 for ω_2 with some defined value of mean ω_{29}, Each generated set checked by the condition 24, and then calculated values $\Phi_i^{(n)}$ for obtained values of $\{\omega_2^{(n)}, \varphi'^{(n)}\}$.

Averaged values have been calculated by the formula:

$$\langle\Phi_i\rangle = \frac{1}{N}\sum_{n=1}^{N}\Phi_i^{(n)} \tag{26}$$

and it's associated error as follows:

$$\triangle\Phi_i = \frac{1}{\sqrt{N(N-1)}} \left\{ \sum_{n=1}^{N} \left(\Phi_i^{(n)}\right)^2 - N\langle\Phi_i\rangle^2 \right\}^{1/2} \tag{27}$$

Here N –number of generated events.

Averaged Stoke's parameters are defined as $\left\langle\xi_i^{(2)}\right\rangle = \langle\Phi_i\rangle / \langle\Phi_0\rangle$ and it's error could be estimated as follows:

From (27) one can see that with probability P confidence interval for Φ_i is:

$$\langle\Phi_i\rangle - t_P \triangle\Phi_i \le \Phi_i \le \langle\Phi_i\rangle + t_P \triangle\Phi_i$$

Here t_P corespondent factor (we have used $P = .95$ and $t_P = 1.96$).

Therefore for ratio $\xi_i^{(2)} = \Phi_i/\Phi_0$ we will have corresponding confidence interval:

$$\left\langle\xi_i^{(2)}\right\rangle - \triangle\xi_i^{(2)} \le \xi_i^{(2)} \le \left\langle\xi_i^{(2)}\right\rangle + \triangle\xi_i^{(2)} \tag{28}$$

where $\triangle\xi_i^{(2)} = \frac{t_P}{\langle\Phi_0\rangle}\left\{\triangle\Phi_i + \left|\left\langle\xi_i^{(2)}\right\rangle\right| \triangle\Phi_0\right\}$.

From (26-28) one can see that by increasing of N may be achieved any desired accuracy of calculation of $\left\langle\xi_i^{(2)}\right\rangle$.

At the numerical calculations we have used following values of parameters, appropriated to SIBERIA-2 facility:

$$\sigma_x = 5.0 \cdot 10^{-4}, \quad \sigma_y = 1.0 \cdot 10^{-4}, \quad \sigma_\omega = 0.05 \cdot \omega_{20}, \quad L = 18m, R = .02m$$

$$\varepsilon_1 = 2.5 GeV, \quad \omega_1 = 2.34 eV$$

3   Results of calculations

From the beginning we should like to consider the case when initial laser photon has pure circular polarization ($\xi_1^{(1)} = \xi_3^{(1)} = 0; \xi_2^{(1)} = 1.0$). Effects of initial electron's polarization will not be considered. From equation (18 - 22) one can see that in this case values Φ_i do not depend on φ' and only source of dispersion of cros section and polarization of outgoing photon is dispersion of measuring ω_2. In the figure 2 are plotted values of outgoing photon averaged circular polarization $\left\langle\xi_2^{(2)}\right\rangle = \langle\Phi_2\rangle / \langle\Phi_0\rangle$ versus mean ω_{20} for the cases when variance $\sigma_\omega = 0$ and $\sigma_\omega = 0.05\ \omega_{20}$. Shown also error bar calculated by formula (28), $N = 10^6$ for each ω_{20}. In the case $\sigma_\omega = 0$ final photon energy is measured exactly and $\omega_2 = \omega_{20}$. Corresponding to this case, the curve of polarization dependence on ω_{20} could be regarded as theoretical curve for dependence of final photon polarization on energy ω_2. One can see that taking into account dispersion of final photon energy measuring σ_ω don't leads to the increasing of final photon polarization dispersion but causes some systematical deviation in the dependence of averaged Stoke's parameter on measured photon energy ω_{20}. This deviation increases when ω_{20} tends to the maximal bound ω_{2max} and attains value about 1.3% for $\sigma_\omega = 0.05\ \omega_{20}$.

In figure 3 are shown results of analogous calculations for value $\left\langle\xi_3^{(2)}\right\rangle$ for the case when initial photon is linear polarized, $\xi_3^{(1)} = 1, \xi_1^{(1)} = \xi_2^{(1)} = 0$. Averaging and estimation

of errors has been provided with help of equations (28) for $N = 10^6$ per each ω_{20}. One can see that effect of deviation of dependence of final photon Stoke's parameters on ω_{20} from theoretical dependence on ω_2 (case $\sigma_\omega = 0$) is observed for the case of linear polarization too. If $\sigma_\omega = 0.05\omega_{20}$ then maximal deviation attains about 0.7%., and if $\sigma_\omega = 0.1\ \omega_{20}$, then maximal deviation is $\sim 3.3\%$.

Mentioned effect points out the necessity of gamma–polarimeter to check the final photon beam polarization parameters at created and existing facilities .

Appendix

Vector $j_\mu^{(1)}$is a unit space-like vector of the form (5):

$$j_\mu^{(1)} = \varepsilon^{\mu\nu\rho\sigma} k_{1\nu} p_{1\rho} k_{2\sigma}/a; \qquad \left(j_\mu^{(1)}\right)^2 = -1. \tag{A.1}$$

By the gauge transformation it may be done a pure space vector, orthogonal to $\overrightarrow{k}_1$:

$$\widetilde{j}_\mu^{(1)} = j_\mu^{(1)} + \beta_1 k_{1\mu}; \quad \widetilde{j}_0^{(1)} =); \widetilde{\overrightarrow{j}}^{(1)} = \overrightarrow{j}_\perp^{(1)}$$

where

$$\overrightarrow{j}_\perp^{(1)} = C_1 \left[\overrightarrow{j}^{(1)} - \overrightarrow{k}_1 \left(\overrightarrow{j}^{(1)} \cdot \overrightarrow{k}_1 \right) \right]; \quad \left(\overrightarrow{j}_\perp^{(1)} \right)^2 = 1, \quad \left(\overrightarrow{j}_\perp^{(1)} \cdot \overrightarrow{k}_1 \right) = 0,$$

$$\left(\overrightarrow{j}^{(1)} \right)_m = C \left\{ \varepsilon^{m0rs} k_{10} p_{1r} k_{2s} + \varepsilon^{mn0s} k_{1n} p_{10} k_{2s} + \varepsilon^{mnr0} k_{1n} p_{1r} k_{20} \right\}_m$$

$$\begin{aligned}
&= C\omega_1\varepsilon_1\omega_2\{-v\left[\overrightarrow{n} \times \overrightarrow{\kappa}_2\right] + \left[\overrightarrow{\kappa}_1 \times \overrightarrow{\kappa}_2\right] - v\left[\overrightarrow{\kappa}_1 \times \overrightarrow{n}\right]\}_m \\
&= C\omega_1\omega_2\varepsilon_1\left[(v\overrightarrow{n} - \overrightarrow{\kappa}_1) \times (v\overrightarrow{n} - \overrightarrow{\kappa}_2)\right]_m;
\end{aligned}$$

$$v = |\overrightarrow{p}_1|/\varepsilon_1 = \sqrt{1 - \gamma^{-2}}; \quad \overrightarrow{n} = \overrightarrow{p}_1/|\overrightarrow{p}_1|; \quad \overrightarrow{\kappa}_1 = \overrightarrow{k}_1/\omega_1; \ \overrightarrow{\kappa}_2 = \overrightarrow{k}_2/\omega_2.$$

That is

$$\overrightarrow{j}_\perp^{(1)} = C_1 \left(\left[(v\overrightarrow{n} - \overrightarrow{\kappa}_1) \times (v\overrightarrow{n} - \overrightarrow{\kappa}_2)\right] - \overrightarrow{\kappa}_1 \left(\overrightarrow{\kappa}_1 \cdot \left[(v\overrightarrow{n} - \overrightarrow{\kappa}_1) \times (v\overrightarrow{n} - \overrightarrow{\kappa}_2)\right] \right) \right)$$

$$= C_1 \left(\left[(v\overrightarrow{n} - \overrightarrow{\kappa}_1) \times (v\overrightarrow{n} - \overrightarrow{\kappa}_2)\right] - \overrightarrow{\kappa}_1 v \left(\overrightarrow{n}\left[\overrightarrow{\kappa}_1 \times \overrightarrow{\kappa}_2\right] \right) \right) \tag{A 2.1}$$

and from $\left| \overrightarrow{i}^{(1)} \right|^2 = 1$:

$$\begin{aligned}
C_1^{-2} &= 1 + v^2 - \left(1 - v^2\right)\left(\overrightarrow{\kappa}_1 \cdot \overrightarrow{\kappa}_2\right)^2 + 2v\left(\overrightarrow{\kappa}_1 \cdot \overrightarrow{\kappa}_2\right) \cdot \\
&\quad \left[\left(\overrightarrow{\kappa}_1 \cdot \overrightarrow{n}\right) + \left(\overrightarrow{\kappa}_2 \cdot \overrightarrow{n}\right) - v\left(\overrightarrow{\kappa}_1 \cdot \overrightarrow{n}\right)\left(\overrightarrow{\kappa}_2 \cdot \overrightarrow{n}\right) - v\right] +
\end{aligned}$$

$$2v\left[v\left(\overrightarrow{\kappa}_1 \cdot \overrightarrow{n}\right)\left(\overrightarrow{\kappa}_2 \cdot \overrightarrow{n}\right) - \left(\overrightarrow{\kappa}_1 \cdot \overrightarrow{n}\right) - \left(\overrightarrow{\kappa}_2 \cdot \overrightarrow{n}\right)\right] \tag{A2.2}$$

Expanding (A2.1, A2.2) into a power series about $\theta_x, \theta_y, \alpha_x, \alpha_y$ (see equation2) we shall get:

$$C_1^{-1} = (1 - \cos\theta_1)\sqrt{(\alpha_x - \theta_x)^2 + (\alpha_y - \theta_y)^2}\left(1 + 0\left(\gamma_{-2}, \alpha_x, \alpha_y, \theta_x, \theta_y\right)\right)$$

9

and with a same accuracy:

$$\vec{j}^{\,(1)}_{\perp} = \vec{e}_x \left[\sin(\varphi') + \frac{\sin^2\theta_1}{1-\cos\theta_1} \cos\varphi_1 \sin(\varphi_1 - \varphi') \right]$$

$$+ \vec{e}_y \left[-\cos\varphi' + \frac{\sin^2\theta_1}{1-\cos\theta_1} \sin\varphi_1 \sin(\varphi_1 - \varphi') \right]$$

$$- \vec{e}_z \sin\theta_1 \sin(\varphi_1 - \varphi'); \tag{A3}$$

where

$$\sin\varphi' = \frac{(\alpha_y - \theta_y)}{\sqrt{(\alpha_x - \theta_x)^2 + (\alpha_y + \theta_y)^2}}; \quad \cos\varphi' = \frac{(\alpha_x - \theta_y)}{\sqrt{(\alpha_x - \theta_x)^2 + (\alpha_y - \theta_y)^2}}$$

In the gauge when both $j_\mu^{(1)}$ and $j_\mu^{(2)}$ are pure space vectors we have

$$\vec{j}^{\,(2)}_{\perp} = \vec{j}^{\,(2)} = \left[\vec{\kappa}_1 \times \vec{j}^{\,(1)}_{\perp} \right]$$
$$= \vec{e}_x \left[\cos\theta_1 \cos\varphi' + (1 + \cos\theta_1) \sin\varphi_1 \sin(\varphi_1 - \varphi') \right]$$

$$+ \vec{e}_y \left[\cos\theta_1 \sin\varphi' + (1 + \cos\theta_1) \cos\varphi_1 \sin(\varphi_1 - \varphi') \right]$$

$$- \vec{e}_z \sin\theta_1 \cos(\varphi_1 - \varphi'). \tag{A4}$$

Analogous calculations for $f_\mu^{(i)}$ give

$$\vec{f}^{\,(1)}_{\perp} = C_2 \left\{ \left[(v\vec{n} - \vec{\kappa}_1) \times (v\vec{n} - \vec{\kappa}_2) \right] - \vec{\kappa}_2 v \left(\vec{n} \left[\vec{\kappa}_1 \times \vec{\kappa}_2 \right] \right) \right\} =$$

$$\vec{e}_x \sin\varphi' - \vec{e}_y \cos\varphi' + \vec{e}_z \cdot 0 \left(\theta_1^2, \theta_2^2, \gamma^{-2} \right) \tag{A5}$$

$$\vec{f}^{\,(2)}_{\perp} = \left[\vec{\kappa}_2 \times \vec{f}^{\,(1)}_{\perp} \right] = \vec{e}_y \cos\varphi' + \vec{e}_x \sin\varphi'. \tag{A6}$$

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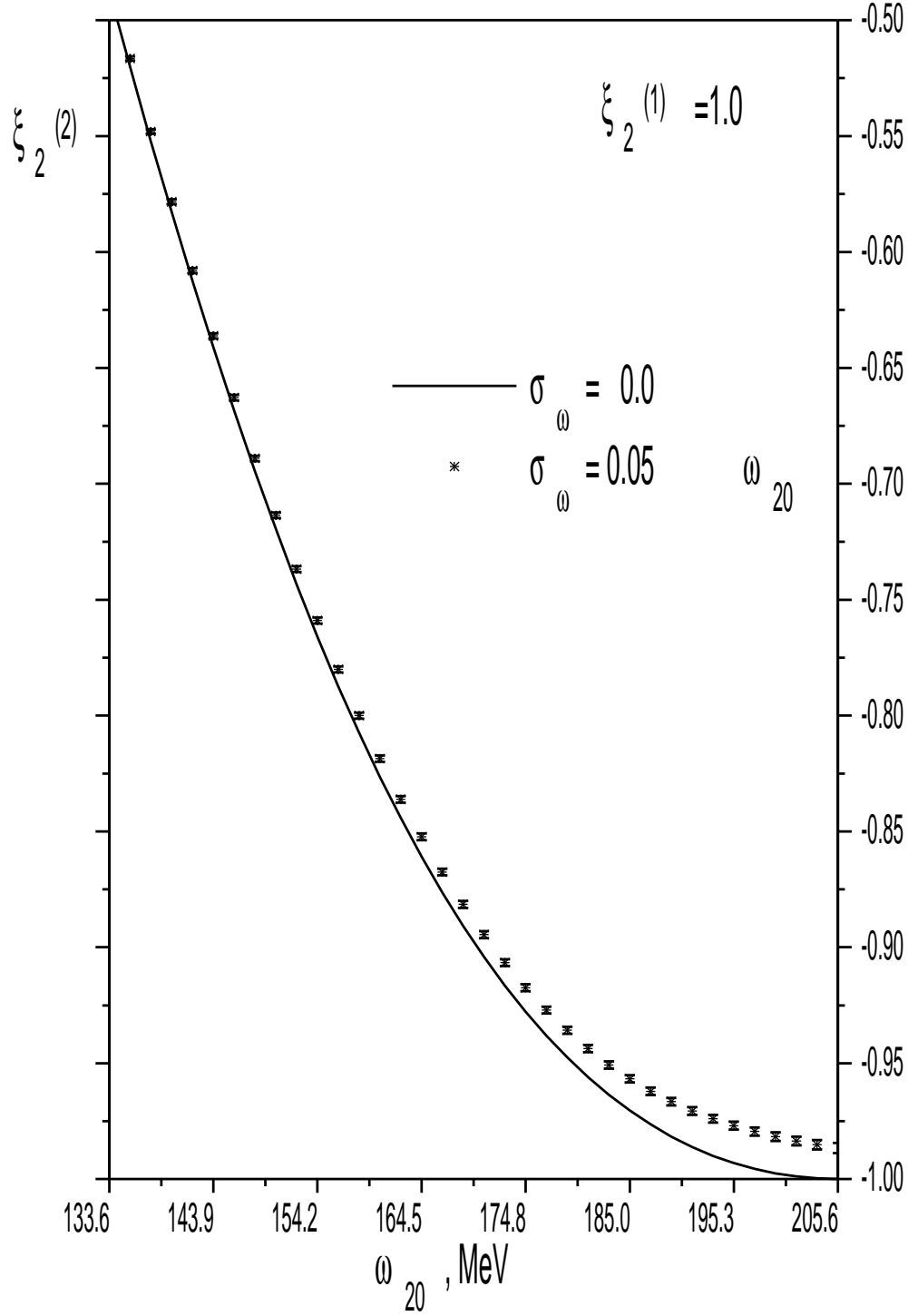[11] R.G.Newton., Scattering Theory of Waves and Particles (McGraw-Hill,1965)

Figure 2: Averaged circular polarization of outgoing photon, $\left\langle \xi_3^{(2)} \right\rangle$ versus ω_{20}. $\xi_2^{(1)} = 0, \xi_1^{(1)} = 0, \xi_3^{(1)} = 1.0$.
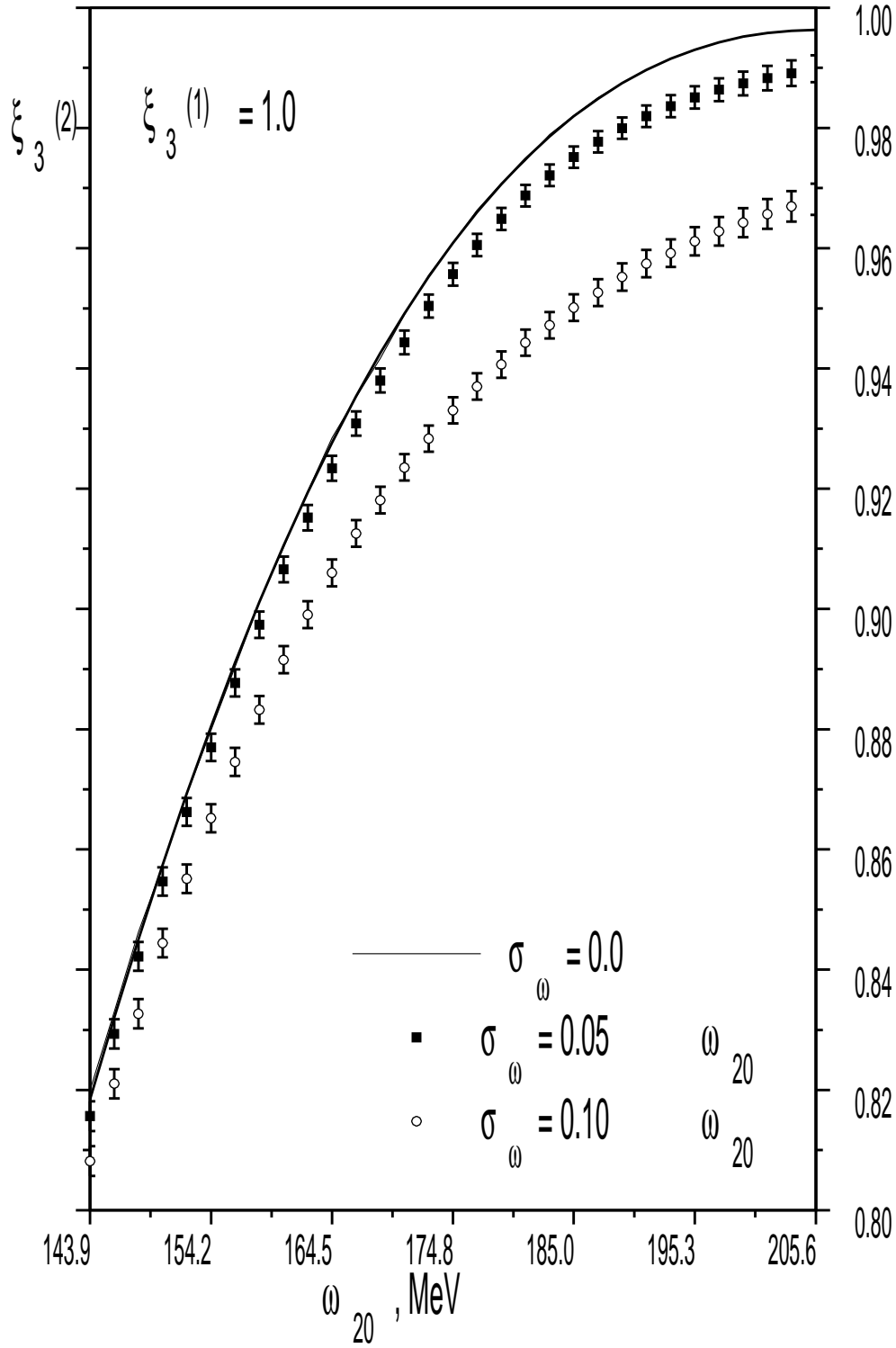Number of simulated events – 10^6 for each value ω_{20}.

Figure 3: Averaged linear polarization of outgoing photon, $\left\langle \xi_3^{(2)} \right\rangle$ versus ω_{20}. $\xi_2^{(1)} = 0, \xi_1^{(1)} = 0, \xi_3^{(1)} = 1.0$.
Number of simulated events – 10^6 for each value ω_{20}.